# Solitary Wave in Water Wave Field



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Biography in Brief



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1963-1997- Disaster Prevention Research Institute, Kyoto University, Japan
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# PREFACE

This work is an extensive work of the scientific research project started in 1960 in Kyoto University. In those years around 1960, we had had suffered by the frequent storm caused by the heavy winds and rain shed, and, the destructive earthquakes accompanied by the huge tsunamis hitting on the coast to devastate the human activity.

The start of the storm in the typhoons were studied in a one-directional model. There was no idea to do anything for protection works at the sudden events induced by the frequent earthquake, especially, by the earthquake accompanied by the tsunami.

It is unfortunate that the early ages in the 21th century, we are in a stage of a quite similar natural condition under unexpected devastative actions of the storms and tsunamis



In 1950s, the scientists and civil engineers even in any university had forced the tentative Operations for the almost fading government functions as much as those who should do promote their research works.

A poor model were introduced for the river flood to discuss that the flood should be solved as a stream though another concept was that the flood should be a kind of waves.

Since then, the scientists promoted in all of the scientific fields under their food shortage. What the activated factors were, for example, the Nobel Prize to Professor Hideki Yukawa and the International Antarctic Scientific Research Project which was promoted by Professor Takeshi Nagata. Almost all of the industries and functions in Japan encouraged at finding the activities of the leading scientists.

Nevertheless, in the early stage of 21st century, the people's human activity seems to be in a low level potentiality.

Now, there is a contribution of the space surveyor “Hayabusa”, and the iPS research project under the promoter, Professor Yamanaka of Kyoto University.

At present, it is necessary to be active for contribution in every scientific fields.

The author now introduces a brief work about “solitary wave”, which had been studied ever once in the last half of the 20st century in Kyoto University.

The author has worked to get some extensive scientific contribution in order to introduce here. He should wish this work introduced here could be an effective impact for those of the scientists and students.

2014 April 1

Director (Retired)
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# Solitary Wave Induced in a Water Surface Wave Field

**Shigehisa Nakamura**
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**Abstract**— A problem on solitary wave induced in a water surface wave field, is introduced for obtaining an analytical solution in a manner of mathematics. By this time, it was recorded offshore at the case of the tsunamis generated by the earthquake in 2011 March 11 in the northwest Pacific. Adding to the above, a hydraulic experiment had been confirmed a couple of solitary waves induced in a field of water surface wave field artificially generated in a basin. Then, a problem is raised to solve in a manner of analytical mathematics for the solitary wave in a water surface wave field under the given conditions.



## 1. INTRODUCTION

This work concerns a certain problem on solitary wave in the water surface wave induced in a water field. This problem could be raised to solve in a manner of applied mathematics for physical science.

By this time, it was recorded offshore at the case of the tsunamis generated by the earthquake on 2011 March 11 in the northwest Pacific by a ocean wave recorder settled on the sea floor for monitoring the ocean water in order to realize the specific property of the ocean water surface waves which have been destructive for the super-tankers' navigation by chance. One of the professors had shown strongly interested in the record of a strange water surface wave at the earthquake under sea. Seismologists had pay a little attention to tsunamis energy in relation to any seismic mechanism.

Adding to the above, a hydraulic experiment had been confirmed a couple of solitary waves induced in a field of water surface wave field artificially generated in a basin.

Now, the problem should be raised to solve in a manner of analytical mathematics for the solitary waves in a water surface wave field under the given condition.

Some part of mechanism for inducing the solitary waves noted above could be realized after obtaining a solution reduced in a mathematical manner in brief.

## 2. SOLITARY WAVE

As for "solitary wave", it can be seen what is the basic for seeing "solitary wave", for example, in the publication by Lamb [1].

In the first edition of "Hydro-Dynamics" written by Lamb [1] must had included the papers presented by McCowan in 1894 [2] and by Korteweg and de-Vries in 1895 [3].

In the waves noted just above in this section, almost all of what are the essential and specific properties of the interested waves named as "solitary waves."

This solitary wave advancing on a straight line in a canal apriori assumed.

The water in the canal is assumed to be isotropic and homogeneous.

Lamb [1] in 1879 started his publication to describe about the water as an idealized fluid for a linearized water motions.

A sinusoidal water surface wave was considered to be a small-amplitude wave first so that the problem was a linear problem.

Nevertheless, the wave discussed by McCowan [2] and Korteweg de-Vries [3] was not any one of the linear waves.

In this case, it was considered only "solitary wave" under a water depth was constant and a width of the canal was uniform.

Lamb [1] had noted about a decay of "solitary wave" though the viscosity was out of his consideration, though he noted about viscosity of fluid in the other chapter later separately.

## 3. TSUNAMI WAVE OBSERVED IN OCEAN

The author has to note that a strange case of the water surface variations observed offshore at the earthquake undersea on 2011 March 11 in the northwest Pacific.

In fact, the strange wave was surely generated and propagating in the ocean. The body force to the water at the earthquake must force to be in motion of the water column rom the surface to the bottom.

The leading wave of tsunamis generated by the earthquake had a sharp spike.

The observed data of the tsunami waves was obtained at only one station settled offshore at that time so that no information was seen about any three-dimensional motion of the water surface patterns.

The sharp spike at the head of the leading tsunami waves forced the seismological scientists' wondering at its dynamical understanding because they had not any bit of proper understanding of water motions at the earthquake even after the seismic event was happened offshore. Then, seismologists had tended to adjust the seismic parameters for finding any reasonable understanding of the processes on the water surface undulations.

## 4. SOLITARY WAVE IN HYDRAULIC EXPERIMENT

Nakamura [4] had ever found "solitary wave" induced in the cyclic water surface wave field.



### 4.1. Monitoring Water Surface Waves

He used a water basin in an arbitrary scaling to see long water wave transform process in the basin after generating a forced cyclic water surface waves.

What was point to see was box-shape block with a slits.

A barrier with a centered slit was arranged to block the incident water waves, so that his intension was to see the wave crest line should be orthogonally cross the barrier.

The barrier width was limited by the wall on both sides of the barrier ends to form a couple of rectangular corners.

The expected incident water wave with a cyclic period was sinusoidal just in front of the wave generator, nevertheless the water wave passed a centered slit for propagating the wave to the barrier just a waves affected to show a Fresnel diffraction at the first slit just in front of the wave generator.

Then, a couple of solitary waves was induced and established at the two corners.

The couple of the solitary waves at the corners moves to maintain each wave form.

Each one of the solitary waves moves between the corner in front of the barrier.

The two solitary waves meet at the center to pass without any wave form change.

Nakamura [4] had tried to confirm whether the couple of the induced waves could be taken as a kind of "solitary wave". Then, he had confirmed the two waves were under some conditions for fitting to McCowan's limit for the solitary wave (refer to [4]).

These two waves moving simply in a cyclic motion along the barrier.

Then, the barrier acted a part of the canal for the solitary wave (for example, [3]).

The energy loss of the couple of the solitary waves was supported by the part of the incident wave energy, even though the decay must be mainly caused energy dissipation after the viscous effect of the water and by the friction on the boundaries as the floor and as the barrier face. Height of the solitary waves decayed out when the incident wave was cut.

The solitary wave in the cyclic waves generated in the water basin was quite similar to the wave form observed at the earthquake noted above section.

It is pity that there was no information to the conditions around the wave gauge settled on the sea floor at the earthquake. Hence, it is hard here to take the observed leading wave with a spike as a solitary wave in a water surface wave field though its wave form pattern is quite similar to that of the couple of the solitary waves induced in a hydraulic base for water surface waves in the field of hydraulics.

### 4.2. Instrumentation

The above solitary wave in the water surface waves in the basin was monitored by an electric and electronic signal transfer system with a mechanical actuator which transfer a designed input function of the signal for generating a water wave in the basin through a metallic pipe in a form of a corresponding oil-pressure signal to force and drive a plunger.

### 4.3. Plunger

The plunger is designed to float in the water basin. Any size and shape could be for a designer's choice, though Nakamura [4] had designed a box-type float made by steel members. It was 7 meters wide, 2 meters long and 0.5 meter high. In order to a stable response to the actuator's forcing oil-pressure under the actuator fixed a cantilever frame work system in a truss structure following the manner in structural engineering standard.

### 4.4. Actuator

The actuator's forcing along its vertical axis transfer its designed forcing to the floating plunger tied to the actuator.

The final target of this system was to generate the expected design water wave in the basin after a transferred output forcing of the oil-pressure controlled by the actuator as they had done for their operations in the field of mechanical engineering.

What was important to drive the system successfully at a water wave generation in the basin, was the expected response of the plunger to be well following to the signal of the output function in a stable manner in motion.

### 4.5. Input Function

The most important part for designing this system was to design the input function for transfer the signal to the actuator generate the expected water waves in the basin.



### 4.6. Feedback

It is usual to control any output signal when an input function is designed. Nevertheless, in Nakamura's case [4], the time lag of the signal was very long so that its delay caused an disagreeable response of water motion in the basin in the early stage of his experiment. Then, the writer had to adjustment of the input function design at first on the bases of the mathematical and dynamical consideration in the fields of engineering technologies. The properly adjusted output function was completed in a finalized form on a disc of the transparent plate made by plastic after his skilled minor adjustment technique.

As for the input function, it was determined with consideration of a lubrication effect of the oil for the actuator and of a viscous effect of a water for experiment in the basin after some minor adjustments for energy balance maintaining during the experiment.

## 5. SOLITARY WAVE IN A WATER WAVE FIELD

Now, it is necessary to clearly describe what problem should be raised.

The spike in the leading wave of the tsunamis in the ocean and a couple of the solitary waves induced in a water surface wave field in a water basin are governed by the equation of motion for the theory in hydrodynamics.

Then, a problem to be raised could be described as that a solitary wave or a couple of the solitary waves should have a solution in a form of an applied mathematics under a certain boundary condition.

When the solution is obtained, it is clarified what mechanism is contributive for the induced solitary wave or waves.

## 6. CONCLUSIONS

A problem is raised for realizing a solitary wave or a couple of the solitary waves could be obtained in a form of a solution in a manner of applied mathematics.

Then, what is mechanism of inducing the solitary wave or a couple of the solitary waves in the water surface wave field.

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# Solitary Wave Induced in a Sinusoidal Water Surface Wave Field of Hydrodynamics

**Shigehisa Nakamura**
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**Abstract**— Monitoring for nonlinear processes of water surface waves is introduced in brief. It was found a kind of nonlinear water surface waves which had a single crest wave with an infinitely long wave in a water tank. This wave was named as solitary wave on water surface in an experiment first. Then, a solitary wave had been led in a mathematical formulation. Another nonlinear wave known as a cnoidal wave was found in a form of cn function mathematically. An extensive experiment in an water tank has led to see a process in which an input function of linear sinusoidal wave induced a nonlinear wave quite similar to the solitary wave.



## 1. INTRODUCTION

This work is concerning to a problem on monitoring of solitary wave on water surface. For this purpose, a historical review is introduced about an experiment of a single crested wave in brief. It was found a kind of nonlinear water surface waves which had a single crest wave with an infinitely long wave in a water tank. This wave was named as solitary wave on the water surface in an experiment first. Then, a solitary wave had been led in a mathematical formulating procedure. Another nonolinear wave known as a cnoidal wave was found in a form of cn function mathematically. An extensive experiment had been led to see a process in which an input function of linear sinusoidal wave induced a nonolinear wave quite similar to the solitary wave.

## 2. LINEAR MODEL OF WATER SURFACE WAVE

At starting to introduce some specific features of nonlinear water surface wave, a brief note is given some note to water waves for a convenience in an hydrodynamic understanding of the present interesting nonlinear waves.

First, assume that water is isotropic for a linear water surface wave model.

This assumption has been introduced even in the publication by Lamb in 1879 [1].

What is essential is the problems on a sinusoidal water surface wave under assuming of a small amplitude as a linear problem of water waves in hydrodynamics.

## 3. NONLINEAR PROCESS OF SOLITARY WAVE

On the other hand, there are various kinds of aperiodic waves.

For example, Lamb in 1879 had introduced about a solitary wave which was called by Scott Russel as seen in "Report on Waves" appeared in British Association Report in 1844 [1]. Following what noted Lamb had noted in 1879, the author could Introduce the work undertaken by Scott Russel as follow.

Scott Russell, in his interesting experimental investigations, was led to pay great attention to a particular type which he called the 'so1itary wave'. This is a wave consisting of a single elevation, of height not necessarily small compared with the depth of the fluid, which, if properly started, may travel for a considerable distance along a uniform canal, with little or no change of type. Waves of depression, of similar relative amplitude, were found not to possess the same character of performance, but to break up into series of shorter waves.

What noted above was given independently by Bousinesquq [2] and Rayleigh (as noted in his publication [3]).

The above note can specify the solitary wave. Then, some note in relation to one of the water waves reduced by a mathematical procedure as what is noted by Lamb in 1879 can be seen as follow.

Russel's solitary type which may be regarded as an extreme case of Russel's solitary type may be regarded as an extreme case of Stokes' oscillatry waves of permanent type, the wave length being great compared with the depth of the canal, so that the widely separated elevations are particularly independent of one another. Now, the methods of approximation employed by Stokes become, however, unsuitable when the wave-length much exceeds the depth; and subsequent investigations of solitary waves of permanent type have proceeded on different lines.

### 3.1. What Lamb Noted

Lamb notes that Rayleigh, treating the problem as one of steady motion, starts virtually the formula

$$\phi + i\psi = F(x+iy) = \exp[iy(d/dx)]F(x), \tag{1}$$

where $F(x)$ is real. The notations $\phi$ and $\psi$ are for stream function and potential velocity.

Lamb's solution of the above differential equation can be written as following, i.e.,

$$\eta = a\,\mathrm{sech}^2[(1/2)(x/b)], \tag{2}$$

where,

$$y - h = \eta, \text{ and, } b^2 = \left[h^2(h+a)/(3a)\right], \tag{3}$$

if the origin of $x$ be taken beneath the summit. The approximations consist in neglecting the fourth power of the ratio $(h+a)/(2b)$. The theory of the solitary waves has been treated by Weinstein in 1926 [1]. Then, it can be seen that

$$c^2 = g(h+a), \tag{4}$$

in the field of the earth's gravity field.

The motion at the outskirts of the solitary wave can be represented by a very simple formula. Lamb in 1987 had communicated from Stokes that McCowan investigated to reduce,

$$c^2 = (g/m)\tan(mh), \tag{5}$$

$$\text{where, } m\alpha = (2/3)\sin^2[m(h+(2/3)a)], \text{ and, } a = \alpha\tan[(1/2)m(h+a)], \tag{6}$$

and, the notations $a$ and $\alpha$ are the maximum elevation above the mean level and a subsidiary constant.

Then, Lamb notes what are specific in the solitary wave. That is to say, the extreme form of the wave when the crest has sharp angle of 120 degree was examined. Adding to that, the limiting value of the ratio $a/h$ was found to be 0.78, in which case the wave-velocity is given by $c^2 = 1.56gh$.

Lamb states as that by a slight modification the investigation of Rayleigh and Boussinesq can be made to give the theory of a system of oscillatory waves of finite height in a canal of limited depth [4].

### 3.2. McCowan and Solitary Wave

McCowan [5] must be a reference in the early age of solitary wave's description.

Assuming a reference water depth $h$, then, a solitary wave $\eta$ can be described in a mathematical form, i.e., as following to Lamb,

$$\eta = a\,\mathrm{sech}^2\left[(1/2)\left(3a/\left(h^3\right)\right)^{1/2} x\right], \tag{7}$$

where, the notation $x$ is the horizontal axis and, the notation $a$ is found in trocoidal wave,

$$x = x_0 a\ \exp(kz_0))\sin(kx_0), \text{ and, } z = z_0 + a\ \exp(kz_0)\cos(kx_0), \tag{8}$$

which is called as Gerstner wave in a consideration of historical background of its finding, as Gerstner reduced theoretically in 1802 though Rankin independently found it in 1863.

As for the small amplitude wave, formulation can be in a linear expression though this solitary wave has to be solved on the basis of nonlinear equation referring Lagrangean technique ia consideration.

## 4. RESONANT COUPLING OF SOLITARY WAVE

Nakamura [6] has found a specific condition can produce a solitary wave in a simple water basin experimentally though a cnoidal wave with cn function was in his interest.

In Nakamura's work [6], he expected a uni-nodal lateral oscillation can be seen first.

The two solitary waves translated to meet at the center of the barrier to increase the maximum height of the water level was about twice of those at the two side comers.

At the center of the barrier, a couple of the solitary wave meets to make a collision each other. After that, a couple of the solitary wave continue the cross motion as if the two waves have never distorted at the collision.

The collision cycle was to coincize to the cycle of to the incident sinusoidal wave as an input function of energy.

So that, a center slit of the barrier must be acted as an energy supply for the two solitary waves to maintain their cross motion between the two side walls during the input function of the sinusoidal wave is repeatedly generated.

When the input function acts only for one cycle of the sinusoidal wave, the couple of the solitary wave becomes to be faint and repeat reflection on the wall of both side up to decay out and disappear the couple of the wave.

This process was seen at the frequency of the input function is satisfying a resonant mode under a certain combination when the boundary condition, bathymetric condition, and energy supply at the center of the barrier are satisfied the resonant mode of the couple of solitary wave in the water basin.

An illustration for various waves found on the water surface is shown by a dot as in Figure 1 [6]. Then, the couple of the solitary wave observed in the water basin fit to the McCowan limit extending an asymptotic line to the case of solitary wave of the water wave theory (a dot in Figure 1).



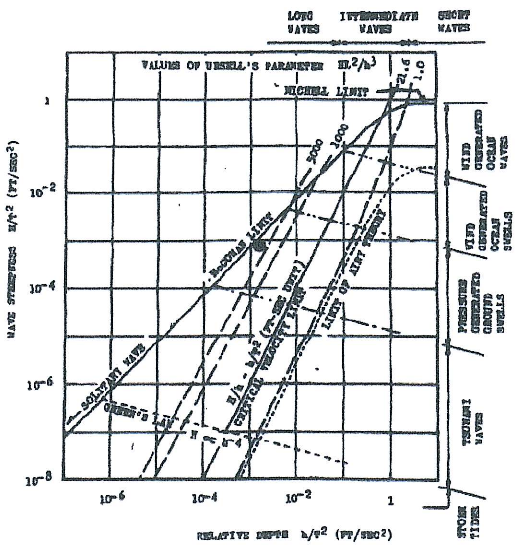


Figure 1: Characteristics of the couple of solitary waves. (1) A dot is for the experimental result. (2) McCowan's limit of breaking wave is demonstrated. (3) McCowan's limit extonding to solitary ware in noticed. [courtesy of ASCE].

This result supports that the cross motion of the couple of solitary wave along the bathymetric line of the specific constant waterdepth crossing to the direction of the input function of sinusoidal wave.

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# Forced Solitary Wave in Water Wave Basin under the Earth's Gravity Field

**Shigehisa Nakamura**
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**Abstract**— Monitoring of solitary wave on water surface is introduced. For this purpose, a water basin for a hydraulic modeling is used. Under a given boundary condition of a hydraulic model, a sinusoidal wave is propagates to pass a slit to diffract just like a case of Fresnel diffraction of optical ray. A specific linkage of boundary conditions, bathymetry, and a generated sinusoidal wave with a specific period is found for producing a solitary water wave in the basin. Some experiments are helpful for basic understanding of a process of solitary wave formation at a generated sinusoidal wave. Then, the author tends to consider whether this solitary wave might suggest that the strange peak observed during the monitoring tsunami waves accompanied by the 2011 earthquake in the northwest Pacific.



## 1. INTRODUCTION

This work is on monitoring of solitary wave on water surface. For this purpose, a water basin for a hydraulic modeling is used. Under a given boundary condition of a hydraulic model, a sinusoidal wave is propagates to pass a slit to diffract just like a case of Fresnel diffraction of optical ray. A specific linkage of boundary conditions, bathymetric condition, and a generated sinusoidal wave with a specific period is found for producing a solitary water wave in the basin. Some experiments are helpful for basic understanding solitary wave formation at a generated sinusoidal wave.

Then, the author tends to consider whether this solitary wave might suggest that the strange peak observed during the monitoring tsunami waves accompanied by the 2011 earthquake in the northwest Pacific.

## 2. SOLITARY WAVE

It is well known that a simple harmonic wave on the water surface can be described as a small amplitude sinusoidal wave.

On the basis of this understanding, a linear formulation can be given generally that the water waves found on the water surface can be described as an ensemble of the waves with various kinds of frequencies and of wave numbers. This means that the waves have a periodic property.

On the other hand, there are various kinds of aperiodic waves.

For example, Lamb [1] had introduced about a solitary wave which was named by Scott Russel in referring to "Report on Waves" appeared in British Association Report in 1844. The details are not introduced here in this work simply as solitary wave can be specified as an aperiodic single peaked wave in experiments Lamb, [1879; 1957].

McCowan [2] must be a reference in the early age of solitary wave's description.

Assuming a reference water depth $h$, then, a solitary wave $\eta$ can be described in a mathematical form, i.e., as following to Lamb,

$$\eta = a\,\mathrm{sech}^2\left[(1/2)(3a/\left(h^3\right))^{1/2}x\right], \tag{1}$$

where, the notation $x$ is the horizontal axis and, the notation $a$ is found in trocoidal wave,

$$x = x_0 a \exp(kz_0)\sin(kx_0), \quad \text{and,} \quad z = z_0 + a\exp(kz_0)\cos(kx_0), \tag{2}$$

which is called as Gerstner wave in a consideration of historical background of its finding, as Gerstner reduced theoretically in 1802 though Rankin independently found it in 1863.

As for the small amplitude wave, formulation can be in a linear expression though this solitary wave has to be solved on the basis of nonlinear equation referring Lagrangean technique in consideration.

## 3. HYDRAULIC MODEL FOR SOLITARY WAVE

Nakamura [3] has found s specific condition can be produce a solitary wave in a sample water basin experimentally.

Nakamura [3] had first aimed to produce a resonant mode in a harbor model or in a bay model by using the water basin in a manner of hydraulics.

His model was 7 m wide and 10 m long. Both of the side part is bounded by a vertical wall made from flat mortar face. The one end is face to the wave generating system. This system was consisted by an actuator of piston type controlled by a hybrid system of analog system and digital system. A plunger floating this end was connected to the actuator, so that the vertical oscillatory motion of the plunger acts as a forced water wave generator.

The actuator is fixed a steel frame work to transfer the signal of the controller to the plunger. Following the signal, the piston and plunger were driven to generate the water waves to propagate into the basin.

The other end of the basin is closed though in his case the bed of the basin is well arranged by a mortal plane in a gentle slope from the plunger side to the other end side. The slope of the bed was 50 cm high to 10 cm long, that is to say, gradient is 1/50.



Inside of the basin, a model shoreline was set at a distance from the front of the plunger. At the distances of 2 meter (Barrier A) and 4 meter (Barrier B), a barrier with a slit of 0.8 meter wide, respectively.

## 4. LINEAR WAVE INPUT FUNCTION

A part of the recorded wave pattern is introduced as shown in Figure 1. To the details, it should be referred to Nakamura's work in 1975.

The illustration in Figure 1 demonstrates that the specific pattern of the response of water in the hydraulic model basin.

The linear sinusoidal input function for the water wave in the basin transfers the wave energy in the basin to induce the resonant mode in the basin though this resonant mode is not so simple.

One of the typical pattern of the wave at the Station 3 is interesting one.

At this station, it can be seen that a cyclic collision of the couple of two solitary waves induced in the basin though separate into a couple of solitary wave pattern without any change of ach wave energy apparently.

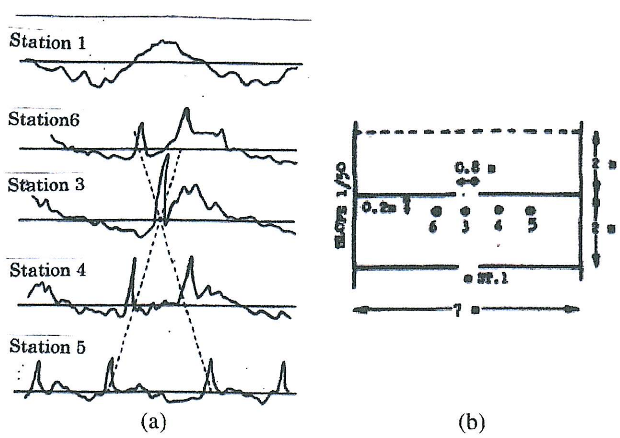


Figure 1: An evolution pattern of a couple of solitary water wave mode in a water basin. (a) (Station-1) An incident transferring passing a slit of barrier A. (Station-6) A transferring sinusoidal wave at the left of barrier B. (Station-3) A collision of the two young solitary waves at B center. (Station-4) A separation of the two growing solitary wave. (Station-5) A matured two solitary waves and a fain sinusoidal wave. (b) Plane of water basin profile. (1) Width is 7 m. (2) Length between two barriers A and B is 2 m. (3) Each slit of barriers A and B is 0.8 m, and, (4) positioning of Stations 1, 6, 3, 4, and 5, respectively.

## 5. INDUCED COUPLE OF NONLINEAR WAVES

The input function looks contributive to force at inducing a couple of the solitary waves in the water basin though it should be reminded that the water in the basin is a kind of the viscous liqids.

It should be reminded that the water is familiar to us, nevertheless nothing of the physical and dynamical properties are understood for us.

We have simply a basic table on the specific properties in physical and chemical phases.

## 6. A LEADING TSUNAMI WAVE WITH A SPIKE

Now, a recent seismic event in 2011 in the northwestern Pacific Ocean had given us a tsunami leading wave with a spike on the wave record at a location just neighbor the seismic epicenter (source of the earthquake).

Its wave form was apparently felt to be quite similar to the wave pattern as seen in the illustration in Figure 1 of this work.

Nevertheless, it should be aware of that the tsunami waves generated at the seismic fault formation on the sea floor is in a kind of transitional processes in a scope of hydrodynamics.

Then, the two waves in the basin and in the ocean could not be any of the same background in hydrodynamics even though those were obtained any advanced electronic recording systems respectively.

A common factor can be found that both of the two waves were produced by a forcedly. That is to say, the wave in the basin was induced by a forcing input function for an actuator and the wave in the ocean was generated at a sudden seismic displacement of the local sea floor.



There is left for a problem to be solved is that a linear input function can be in a position for any nonlinear wave, for example, a solitary wave in a model.

## 7. NUMERICAL MODELING

As far as the author concern, the author has never heard of any numerical modeling of the couple of solitary waves in a basin.

Almost all of the seismologists missed to pay any attention to the tsunami offshore in the ocean though noted the above section.

Then, the author takes it pity but he has to write that the scientists have to wait for a numerical modeling which gives a more reasonable solution in an advanced technique in a time not later than any hazardous event happens.

## 8. CONCLUSIONS

The author noted about monitoring of solitary wave on water surface in introduced. An induced resonant couple of solitary waves in a water surface wave in a open water basin for modeling was a trigger of the problem to see whether a linear input function can contributive for a nonlinear process in hydraulics and also in hydrodynamics.

There should be required for the purpose any one of some techniques, for example, a simple approximation, or, asymptotic concept for mathematical singularity, or relaxation for discontinuity, or, cut-off of high frequency band or high wave number constituents at digitizing the continuous functions.

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APPENDICES

# Solitary Wave Induced in a Water Surface Wave Field

**Shigehisa Nakamura**
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**Abstract**— A problem on solitary wave induced in a water surface wave field, is introduced for obtaining an analytical solution in a manner of mathematics. By this time, it was recorded offshore at the case of the tsunamis generated by the earthquake in 2011 March 11 in the northwest Pacific. Adding to the above, a hydraulic experiment had been confirmed a couple of solitary waves induced in a field of water surface wave field artificially generated in a basin. Then, a problem is raised to solve in a manner of analytical mathematics for the solitary wave in a water surface wave field under the given conditions.



## 1. INTRODUCTION

This work concerns a certain problem on solitary wave in the water surface wave induced in a water field. This problem could be raised to solve in a manner of applied mathematics for physical science.

By this time, it was recorded offshore at the case of the tsunamis generated by the earthquake on 2011 March 11 in the northwest Pacific by a ocean wave recorder settled on the sea floor for monitoring the ocean water in order to realize the specific property of the ocean water surface waves which have been destructive for the super-tankers' navigation by chance. One of the professors had shown strongly interested in the record of a strange water surface wave at the earthquake under sea. Seismologists had pay a little attention to tsunamis energy in relation to any seismic mechanism.

Adding to the above, a hydraulic experiment had been confirmed a couple of solitary waves induced in a field of water surface wave field artificially generated in a basin.

Now, the problem should be raised to solve in a manner of analytical mathematics for the solitary waves in a water surface wave field under the given condition.

Some part of mechanism for inducing the solitary waves noted above could be realized after obtaining a solution reduced in a mathematical manner in brief.

## 2. SOLITARY WAVE

As for "solitary wave", it can be seen what is the basic for seeing "solitary wave", for example, in the publication by Lamb [1].

In the first edition of "Hydro-Dynamics" written by Lamb [1] must had included the papers presented by McCowan in 1894 [2] and by Korteweg and de-Vries in 1895 [3].

In the waves noted just above in this section, almost all of what are the essential and specific properties of the interested waves named as "solitary waves."

This solitary wave advancing on a straight line in a canal apriori assumed.

The water in the canal is assumed to be isotropic and homogeneous.

Lamb [1] in 1879 started his publication to describe about the water as an idealized fluid for a linearized water motions.

A sinusoidal water surface wave was considered to be a small-amplitude wave first so that the problem was a linear problem.

Nevertheless, the wave discussed by McCowan [2] and Korteweg de-Vries [3] was not any one of the linear waves.

In this case, it was considered only "solitary wave" under a water depth was constant and a width of the canal was uniform.

Lamb [1] had noted about a decay of "solitary wave" though the viscosity was out of his consideration, though he noted about viscosity of fluid in the other chapter later separately.

## 3. TSUNAMI WAVE OBSERVED IN OCEAN

The author has to note that a strange case of the water surface variations observed offshore at the earthquake undersea on 2011 March 11 in the northwest Pacific.

In fact, the strange wave was surely generated and propagating in the ocean. The body force to the water at the earthquake must force to be in motion of the water column rom the surface to the bottom.

The leading wave of tsunamis generated by the earthquake had a sharp spike.

The observed data of the tsunami waves was obtained at only one station settled offshore at that time so that no information was seen about any three-dimensional motion of the water surface patterns.

The sharp spike at the head of the leading tsunami waves forced the seismological scientists' wondering at its dynamical understanding because they had not any bit of proper understanding of water motions at the earthquake even after the seismic event was happened offshore. Then, seismologists had tended to adjust the seismic parameters for finding any reasonable understanding of the processes on the water surface undulations.

## 4. SOLITARY WAVE IN HYDRAULIC EXPERIMENT

Nakamura [4] had ever found "solitary wave" induced in the cyclic water surface wave field.

### 4.1. Monitoring Water Surface Waves



He used a water basin in an arbitrary scaling to see long water wave transform process in the basin after generating a forced cyclic water surface waves.

What was point to see was box-shape block with a slits.

A barrier with a centered slit was arranged to block the incident water waves, so that his intension was to see the wave crest line should be orthogonally cross the barrier.

The barrier width was limited by the wall on both sides of the barrier ends to form a couple of rectangular corners.

The expected incident water wave with a cyclic period was sinusoidal just in front of the wave generator, nevertheless the water wave passed a centered slit for propagating the wave to the barrier just a waves affected to show a Fresnel diffraction at the first slit just in front of the wave generator.

Then, a couple of solitary waves was induced and established at the two corners.

The couple of the solitary waves at the corners moves to maintain each wave form.

Each one of the solitary waves moves between the corner in front of the barrier.

The two solitary waves meet at the center to pass without any wave form change.

Nakamura [4] had tried to confirm whether the couple of the induced waves could be taken as a kind of "solitary wave". Then, he had confirmed the two waves were under some conditions for fitting to McCowan's limit for the solitary wave (refer to [4]).

These two waves moving simply in a cyclic motion along the barrier.

Then, the barrier acted a part of the canal for the solitary wave (for example, [3]).

The energy loss of the couple of the solitary waves was supported by the part of the incident wave energy, even though the decay must be mainly caused energy dissipation after the viscous effect of the water and by the friction on the boundaries as the floor and as the barrier face. Height of the solitary waves decayed out when the incident wave was cut.

The solitary wave in the cyclic waves generated in the water basin was quite similar to the wave form observed at the earthquake noted above section.

It is pity that there was no information to the conditions around the wave gauge settled on the sea floor at the earthquake. Hence, it is hard here to take the observed leading wave with a spike as a solitary wave in a water surface wave field though its wave form pattern is quite similar to that of the couple of the solitary waves induced in a hydraulic base for water surface waves in the field of hydraulics.

### 4.2. Instrumentation

The above solitary wave in the water surface waves in the basin was monitored by an electric and electronic signal transfer system with a mechanical actuator which transfer a designed input function of the signal for generating a water wave in the basin through a metallic pipe in a form of a corresponding oil-pressure signal to force and drive a plunger.

### 4.3. Plunger

The plunger is designed to float in the water basin. Any size and shape could be for a designer's choice, though Nakamura [4] had designed a box-type float made by steel members. It was 7 meters wide, 2 meters long and 0.5 meter high. In order to a stable response to the actuator's forcing oil-pressure under the actuator fixed a cantilever frame work system in a truss structure following the manner in structural engineering standard.

### 4.4. Actuator

The actuator's forcing along its vertical axis transfer its designed forcing to the floating plunger tied to the actuator.

The final target of this system was to generate the expected design water wave in the basin after a transferred output forcing of the oil-pressure controlled by the actuator as they had done for their operations in the field of mechanical engineering.

What was important to drive the system successfully at a water wave generation in the basin, was the expected response of the plunger to be well following to the signal of the output function in a stable manner in motion.

### 4.5. Input Function

The most important part for designing this system was to design the input function for transfer the signal to the actuator generate the expected water waves in the basin.

### 4.6. Feedback



It is usual to control any output signal when an input function is designed. Nevertheless, in Nakamura's case [4], the time lag of the signal was very long so that its delay caused an disagreeable response of water motion in the basin in the early stage of his experiment. Then, the writer had to adjustment of the input function design at first on the bases of the mathematical and dynamical consideration in the fields of engineering technologies. The properly adjusted output function was completed in a finalized form on a disc of the transparent plate made by plastic after his skilled minor adjustment technique.

As for the input function, it was determined with consideration of a lubrication effect of the oil for the actuator and of a viscous effect of a water for experiment in the basin after some minor adjustments for energy balance maintaining during the experiment.

## 5. SOLITARY WAVE IN A WATER WAVE FIELD

Now, it is necessary to clearly describe what problem should be raised.

The spike in the leading wave of the tsunamis in the ocean and a couple of the solitary waves induced in a water surface wave field in a water basin are governed by the equation of motion for the theory in hydrodynamics.

Then, a problem to be raised could be described as that a solitary wave or a couple of the solitary waves should have a solution in a form of an applied mathematics under a certain boundary condition.

When the solution is obtained, it is clarified what mechanism is contributive for the induced solitary wave or waves.

## 6. CONCLUSIONS

A problem is raised for realizing a solitary wave or a couple of the solitary waves could be obtained in a form of a solution in a manner of applied mathematics.

Then, what is mechanism of inducing the solitary wave or a couple of the solitary waves in the water surface wave field.

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# Solitary Wave Induced in a Sinusoidal Water Surface Wave Field of Hydrodynamics

**Shigehisa Nakamura**
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**Abstract**— Monitoring for nonlinear processes of water surface waves is introduced in brief. It was found a kind of nonlinear water surface waves which had a single crest wave with an infinitely long wave in a water tank. This wave was named as solitary wave on water surface in an experiment first. Then, a solitary wave had been led in a mathematical formulation. Another nonlinear wave known as a cnoidal wave was found in a form of cn function mathematically. An extensive experiment in an water tank has led to see a process in which an input function of linear sinusoidal wave induced a nonlinear wave quite similar to the solitary wave.



## 1. INTRODUCTION

This work is concerning to a problem on monitoring of solitary wave on water surface. For this purpose, a historical review is introduced about an experiment of a single crested wave in brief. It was found a kind of nonlinear water surface waves which had a single crest wave with an infinitely long wave in a water tank. This wave was named as solitary wave on the water surface in an experiment first. Then, a solitary wave had been led in a mathematical formulating procedure. Another nonolinear wave known as a cnoidal wave was found in a form of cn function mathematically. An extensive experiment had been led to see a process in which an input function of linear sinusoidal wave induced a nonolinear wave quite similar to the solitary wave.

## 2. LINEAR MODEL OF WATER SURFACE WAVE

At starting to introduce some specific features of nonlinear water surface wave, a brief note is given some note to water waves for a convenience in an hydrodynamic understanding of the present interesting nonlinear waves.

First, assume that water is isotropic for a linear water surface wave model.

This assumption has been introduced even in the publication by Lamb in 1879 [1].

What is essential is the problems on a sinusoidal water surface wave under assuming of a small amplitude as a linear problem of water waves in hydrodynamics.

## 3. NONLINEAR PROCESS OF SOLITARY WAVE

On the other hand, there are various kinds of aperiodic waves.

For example, Lamb in 1879 had introduced about a solitary wave which was called by Scott Russel as seen in "Report on Waves" appeared in British Association Report in 1844 [1]. Following what noted Lamb had noted in 1879, the author could Introduce the work undertaken by Scott Russel as follow.

Scott Russell, in his interesting experimental investigations, was led to pay great attention to a particular type which he called the 'so1itary wave'. This is a wave consisting of a single elevation, of height not necessarily small compared with the depth of the fluid, which, if properly started, may travel for a considerable distance along a uniform canal, with little or no change of type. Waves of depression, of similar relative amplitude, were found not to possess the same character of performance, but to break up into series of shorter waves.

What noted above was given independently by Bousinesquq [2] and Rayleigh (as noted in his publication [3]).

The above note can specify the solitary wave. Then, some note in relation to one of the water waves reduced by a mathematical procedure as what is noted by Lamb in 1879 can be seen as follow.

Russel's solitary type which may be regarded as an extreme case of Russel's solitary type may be regarded as an extreme case of Stokes' oscillatry waves of permanent type, the wave length being great compared with the depth of the canal, so that the widely separated elevations are particularly independent of one another. Now, the methods of approximation employed by Stokes become, however, unsuitable when the wave-length much exceeds the depth; and subsequent investigations of solitary waves of permanent type have proceeded on different lines.

### 3.1. What Lamb Noted

Lamb notes that Rayleigh, treating the problem as one of steady motion, starts virtually the formula

$$\phi + i\psi = F(x + iy) = \exp[iy(d/dx)]F(x), \tag{1}$$

where $F(x)$ is real. The notations $\phi$ and $\psi$ are for stream function and potential velocity.

Lamb's solution of the above differential equation can be written as following, i.e.,

$$\eta = a\,\mathrm{sech}^2[(1/2)(x/b)], \tag{2}$$

where,

$$y - h = \eta, \text{ and, } b^2 = \left[h^2(h + a)/(3a)\right], \tag{3}$$

if the origin of $x$ be taken beneath the summit. The approximations consist in neglecting the fourth power of the ratio $(h + a)/(2b)$. The theory of the solitary waves has been treated by Weinstein in [1]. Then, it can be seen that



$$c^2 = g(h + a), \tag{4}$$

in the field of the earth's gravity field.

The motion at the outskirts of the solitary wave can be represented by a very simple formula. Lamb in 1987 had communicated from Stokes that McCowan investigated to reduce,

$$c^2 = (g/m)\tan(mh), \tag{5}$$

$$\text{where, } m\alpha = (2/3)\sin^2[m(h + (2/3)a)], \text{ and, } a = \alpha\tan[(1/2)m(h + a)], \tag{6}$$

and, the notations $a$ and $\alpha$ are the maximum elevation above the mean level and a subsidiary constant.

Then, Lamb notes what are specific in the solitary wave. That is to say, the extreme form of the wave when the crest has sharp angle of 120 degree was examined. Adding to that, the limiting value of the ratio $a/h$ was found to be 0.78, in which case the wave-velocity is given by $c^2 = 1.56gh$.

Lamb states as that by a slight modification the investigation of Rayleigh and Boussinesq can be made to give the theory of a system of oscillatory waves of finite height in a canal of limited depth [4].

### 3.2. McCowan and Solitary Wave

McCowan [5] must be a reference in the early age of solitary wave's description.

Assuming a reference water depth $h$, then, a solitary wave $\eta$ can be described in a mathematical form, i.e., as following to Lamb,

$$\eta = a\,\mathrm{sech}^2\left[(1/2)\left(3a/\left(h^3\right)\right)^{1/2} x\right], \tag{7}$$

where, the notation $x$ is the horizontal axis and, the notation $a$ is found in trocoidal wave,

$$x = x_0 a\ \exp(kz_0))\sin(kx_0), \text{ and, } z = z_0 + a\ \exp(kz_0)\cos(kx_0), \tag{8}$$

which is called as Gerstner wave in a consideration of historical background of its finding, as Gerstner reduced theoretically in 1802 though Rankin independently found it in 1863.

As for the small amplitude wave, formulation can be in a linear expression though this solitary wave has to be solved on the basis of nonlinear equation referring Lagrangean technique ia consideration.

## 4. RESONANT COUPLING OF SOLITARY WAVE

Nakamura [6] has found a specific condition can produce a solitary wave in a simple water basin experimentally though a cnoidal wave with cn function was in his interest.

In Nakamura's work [6], he expected a uni-nodal lateral oscillation can be seen first.

The two solitary waves translated to meet at the center of the barrier to increase the maximum height of the water level was about twice of those at the two side comers.

At the center of the barrier, a couple of the solitary wave meets to make a collision each other. After that, a couple of the solitary wave continue the cross motion as if the two waves have never distorted at the collision.

The collision cycle was to coincize to the cycle of to the incident sinusoidal wave as an input function of energy.

So that, a center slit of the barrier must be acted as an energy supply for the two solitary waves to maintain their cross motion between the two side walls during the input function of the sinusoidal wave is repeatedly generated.

When the input function acts only for one cycle of the sinusoidal wave, the couple of the solitary wave becomes to be faint and repeat reflection on the wall of both side up to decay out and disappear the couple of the wave.

This process was seen at the frequency of the input function is satisfying a resonant mode under a certain combination when the boundary condition, bathymetric condition, and energy supply at the center of the barrier are satisfied the resonant mode of the couple of solitary wave in the water basin.

An illustration for various waves found on the water surface is shown by a dot as in Figure 1 [6]. Then, the couple of the solitary wave observed in the water basin fit to the McCowan limit extending symptotic line to the case of solitary wave of the water wave theory (a dot in Figure 1).



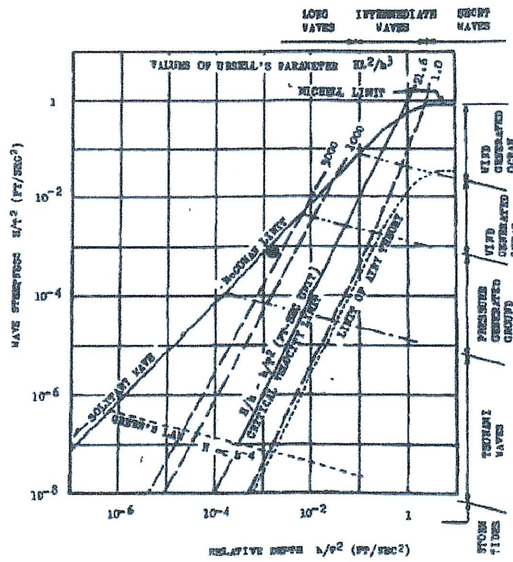


Figure 1: Characteristics of the couple of solitary waves. (1) A dot is for the experimental result. (2) McCowan's limit of breaking wave is demonstrated. (3) McCowan's limit extonding to solitary ware in noticed. [courtesy of ASCE].

This result supports that the cross motion of the couple of solitary wave along the bathymetric line of the specific constant waterdepth crossing to the direction of the input function of sinusoidal wave.

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# Forced Solitary Wave in Water Wave Basin under the Earth's Gravity Field

**Shigehisa Nakamura**
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**Abstract**— Monitoring of solitary wave on water surface is introduced. For this purpose, a water basin for a hydraulic modeling is used. Under a given boundary condition of a hydraulic model, a sinusoidal wave is propagates to pass a slit to diffract just like a case of Fresnel diffraction of optical ray. A specific linkage of boundary conditions, bathymetry, and a generated sinusoidal wave with a specific period is found for producing a solitary water wave in the basin. Some experiments are helpful for basic understanding of a process of solitary wave formation at a generated sinusoidal wave. Then, the author tends to consider whether this solitary wave might suggest that the strange peak observed during the monitoring tsunami waves accompanied by the 2011 earthquake in the northwest Pacific.



## 1. INTRODUCTION

This work is on monitoring of solitary wave on water surface. For this purpose, a water basin for a hydraulic modeling is used. Under a given boundary condition of a hydraulic model, a sinusoidal wave is propagates to pass a slit to diffract just like a case of Fresnel diffraction of optical ray. A specific linkage of boundary conditions, bathymetric condition, and a generated sinusoidal wave with a specific period is found for producing a solitary water wave in the basin. Some experiments are helpful for basic understanding solitary wave formation at a generated sinusoidal wave.

Then, the author tends to consider whether this solitary wave might suggest that the strange peak observed during the monitoring tsunami waves accompanied by the 2011 earthquake in the northwest Pacific.

## 2. SOLITARY WAVE

It is well known that a simple harmonic wave on the water surface can be described as a small amplitude sinusoidal wave.

On the basis of this understanding, a linear formulation can be given generally that the water waves found on the water surface can be described as an ensemble of the waves with various kinds of frequencies and of wave numbers. This means that the waves have a periodic property.

On the other hand, there are various kinds of aperiodic waves.

For example, Lamb [1] had introduced about a solitary wave which was named by Scott Russel in referring to "Report on Waves" appeared in British Association Report in 1844. The details are not introduced here in this work simply as solitary wave can be specified as an aperiodic single peaked wave in experiments Lamb, [1879; 1957].

McCowan [2] must be a reference in the early age of solitary wave's description.

Assuming a reference water depth $h$, then, a solitary wave $\eta$ can be described in a mathematical form, i.e., as following to Lamb,

$$\eta = a\,\mathrm{sech}^2\left[(1/2)(3a/\left(h^3\right))^{1/2}x\right], \tag{1}$$

where, the notation $x$ is the horizontal axis and, the notation $a$ is found in trocoidal wave,

$$x = x_0 a \exp(kz_0)\sin(kx_0), \quad \text{and}, \quad z = z_0 + a\exp(kz_0)\cos(kx_0), \tag{2}$$

which is called as Gerstner wave in a consideration of historical background of its finding, as Gerstner reduced theoretically in 1802 though Rankin independently found it in 1863.

As for the small amplitude wave, formulation can be in a linear expression though this solitary wave has to be solved on the basis of nonlinear equation referring Lagrangean technique in consideration.

## 3. HYDRAULIC MODEL FOR SOLITARY WAVE

Nakamura [3] has found s specific condition can be produce a solitary wave in a sample water basin experimentally.

Nakamura [3] had first aimed to produce a resonant mode in a harbor model or in a bay model by using the water basin in a manner of hydraulics.

His model was 7 m wide and 10 m long. Both of the side part is bounded by a vertical wall made from flat mortar face. The one end is face to the wave generating system. This system was consisted by an actuator of piston type controlled by a hybrid system of analog system and digital system. A plunger floating this end was connected to the actuator, so that the vertical oscillatory motion of the plunger acts as a forced water wave generator.

The actuator is fixed a steel frame work to transfer the signal of the controller to the plunger. Following the signal, the piston and plunger were driven to generate the water waves to propagate into the basin.



The other end of the basin is closed though in his case the bed of the basin is well arranged by ortal plane in a gentle slope from the plunger side to the other end side. The slope of the bed was 50 cm high to 10 cm long, that is to say, gradient is 1/50.

Inside of the basin, a model shoreline was set at a distance from the front of the plunger. At the distances of 2 meter (Barrier A) and 4 meter (Barrier B), a barrier with a slit of 0.8 meter wide, respectively.

## 4. LINEAR WAVE INPUT FUNCTION

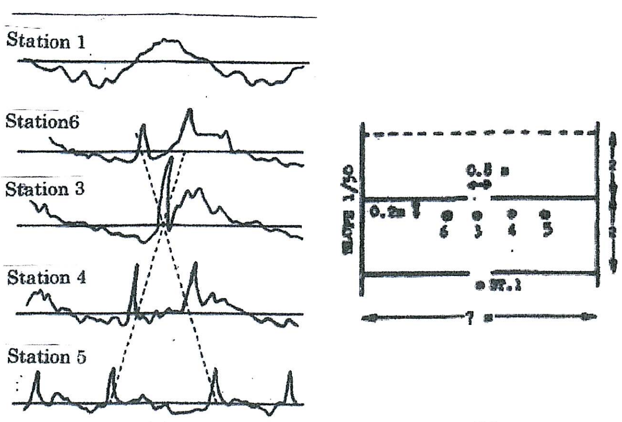
A part of the recorded wave pattern is introduced as shown in Figure 1. To the details, it should be referred to Nakamura's work in 1975.

The illustration in Figure 1 demonstrates that the specific pattern of the response of water in the hydraulic model basin.

The linear sinusoidal input function for the water wave in the basin transfers the wave energy in the basin to induce the resonant mode in the basin though this resonant mode is not so simple.

One of the typical pattern of the wave at the Station 3 is interesting one.

At this station, it can be seen that a cyclic collision of the couple of two solitary waves induced in the basin though separate into a couple of solitary wave pattern without any change of ach wave energy apparently.




(a) (b)

Figure 1: An evolution pattern of a couple of solitary water wave mode in a water basin. (a) (Station-1) An incident transferring passing a slit of barrier A. (Station-6) A transferring sinusoidal wave at the left of barrier B. (Station-3) A collision of the two young solitary waves at B center. (Station-4) A separation of the two growing solitary wave. (Station-5) A matured two solitary waves and a fain sinusoidal wave. (b) Plane of water basin profile. (1) Width is 7 m. (2) Length between two barriers A and B is 2 m. (3) Each slit of barriers A and B is 0.8 m, and, (4) positioning of Stations 1, 6, 3, 4, and 5, respectively.

## 5. INDUCED COUPLE OF NONLINEAR WAVES

The input function looks contributive to force at inducing a couple of the solitary waves in the water basin though it should be reminded that the water in the basin is a kind of the viscous liqids.

It should be reminded that the water is familiar to us, nevertheless nothing of the physical and dynamical properties are understood for us.

We have simply a basic table on the specific properties in physical and chemical phases.

## 6. A LEADING TSUNAMI WAVE WITH A SPIKE

Now, a recent seismic event in 2011 in the northwestern Pacific Ocean had given us a tsunami leading wave with a spike on the wave record at a location just neighbor the seismic epicenter (source of the earthquake).

Its wave form was apparently felt to be quite similar to the wave pattern as seen in the illustration in Figure 1 of this work.

Nevertheless, it should be aware of that the tsunami waves generated at the seismic fault formation on the sea floor is in a kind of transitional processes in a scope of hydrodynamics.

Then, the two waves in the basin and in the ocean could not be any of the same background in hydrodynamics even though those were obtained any advanced electronic recording systems respectively.



A common factor can be found that both of the two waves were produced by a forcedly. That is to say, the wave in the basin was induced by a forcing input function for an actuator and the wave in the ocean was generated at a sudden seismic displacement of the local sea floor.

There is left for a problem to be solved is that a linear input function can be in a position for any nonlinear wave, for example, a solitary wave in a model.

## 7. NUMERICAL MODELING

As far as the author concern, the author has never heard of any numerical modeling of the couple of solitary waves in a basin.

Almost all of the seismologists missed to pay any attention to the tsunami offshore in the ocean though noted the above section.

Then, the author takes it pity but he has to write that the scientists have to wait for a numerical modeling which gives a more reasonable solution in an advanced technique in a time not later than any hazardous event happens.

## 8. CONCLUSIONS

The author noted about monitoring of solitary wave on water surface in introduced. An induced resonant couple of solitary waves in a water surface wave in a open water basin for modeling was a trigger of the problem to see whether a linear input function can contributive for a nonlinear process in hydraulics and also in hydrodynamics.

There should be required for the purpose any one of some techniques, for example, a simple approximation, or, asymptotic concept for mathematical singularity, or relaxation for discontinuity, or, cut-off of high frequency band or high wave number constituents at digitizing the continuous functions.

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